ROLLING MOTION OF A RIGID OBJECT

1.) Show the steps in the detailed derivation (including the use of the P.A.T.) of the equation below for total kinetic energy of this sphere rolling down the incline w/o slipping.

\[ K = \frac{1}{2} \left[ \frac{I_{CM}}{R^2} + m \right] v_{cm}^2 \]

Mark with a small x the axis of rotation of the sphere as it rolls down the incline.

2.) A ring of mass 4.00 kg, inner radius 8.00 cm, and outer radius 12.00 cm rolls (without slipping) up an inclined plane that makes an angle of \( \theta = 30^\circ \). At the moment the ring is at position \( x = 2.50 \) m up the plane, its speed is 3.00 m/s. The ring continues up the plane for some additional distance and then rolls back down. It does not roll off the top end. How far up the plane does it go?
THE VECTOR PRODUCT AND TORQUE

3.) \( \mathbf{k} \times \mathbf{k} = \) \( \mathbf{-j} \times \mathbf{i} = \) \( \mathbf{i} \times \mathbf{-j} = \)

4.) Show the determinant work needed to determine the torque resulting from the following force acting on a rigid body at the radius \( \mathbf{r} \) from the pivot.

\[
\mathbf{r} = (4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) \text{m} \quad \mathbf{F} = (6\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}) \text{N}
\]

5.) Write the three laws (equations) of conservation for an isolated system:

6.) Starting with \( \sum \mathbf{\tau} = \mathbf{r} \times \sum \mathbf{F} \) derive the relationship between \( \sum \mathbf{\tau} \) and angular momentum.

7.) Show the steps needed to get from the general formula for angular momentum to the specific formula for angular momentum of a rotating rigid body (with a moment of inertia \( \mathbf{I} \))

Start:

\[
\mathbf{L} = \mathbf{r} \times \mathbf{p}
\]

\[
\mathbf{L} = \mathbf{I} \mathbf{\dot{\omega}}
\]

8a.) What is the Schluup short cut to finding angular momentum? 

8b.) Draw an example of a situation where it comes in handy:
9.) A particle of mass \( m \) is shot with an initial velocity \( v_0 \) and makes an angle \( \theta \) with the horizontal, as shown in the figure below. The particle moves in the gravitational field of the Earth. Find the angular momentum of the particle about the origin when the particle is at the highest point of the trajectory. (Your answer should involve the symbols \( m, \theta, g, v_0 \))

Hint: You probably want to use the Schluup short cut here.

Required drawing:

![Diagram](image)

10.) A fireman clings to a vertical ladder and directs the nozzle of a hose horizontally toward a burning building. The rate of water flow is 500 liters/minute, and the nozzle speed is 15.6 m/s. The hose passes between the fireman’s feet, which are 1.30 m vertically directly below the nozzle. Choose the origin to be inside the hose between the fireman’s feet. What torque must the fireman exert on the hose?

11.) A sphere of mass \( m_1 \) and a block of mass \( m_2 \) are connected by a light cord that passes over a pulley \((m_p)\) as shown in the fig. below. The radius of the pulley is \( R \), and the moment of inertia about its axle is \( I \). The block slides on a frictionless, horizontal surface. *Using the concepts of angular momentum and torque*, find an expression for the linear acceleration of the two objects in terms of \( m_1, m_2, m_p, \) and \( g \).
12.) A particle with a mass of \textbf{0.500 kg} is attached to the \textbf{100 cm} mark of a meter stick with a mass of \textbf{0.200 kg}. The meter stick rotates on a horizontal, frictionless table with an angular speed of \textbf{3.00 s}^{-1}. Calculate the angular momentum of the system when the stick is pivoted about an axis perpendicular to the table through the \textbf{0 cm} mark.

\textbf{Required labeled Drawing:}

13.) A particular star rotates with a period of \textbf{40 days} about an axis through its center. After the star undergoes a supernova explosion, the stellar core, which has a radius of \textbf{5.0 \times 10^4 km}, collapses into a neutron star of radius \textbf{4.0 km}. Determine the period of rotation of the neutron star in milliseconds.
14.) A 50.0 kg woman stands at the rim of a horizontal turntable having a moment of inertia of 500 kg·m² and a radius of 3.00 m. The turntable is initially at rest and is free to rotate about a frictionless, vertical axis through its center. The woman then starts walking around the rim clockwise (as viewed from above the system) at a constant speed of 2.00 m/s relative to the earth. In what direction and at what angular speed does the turntable rotate?

15.) Fill in the following table which shows data taken from the stick and disk system as shown in the figure below. The disk has a mass of 3.0 kg. The stick has a mass of 1.5 kg. The disk and stick are on nearly frictionless ice. Assume the collision is elastic. The moment of inertia of the stick about its center of mass is 4.50 kg·m².

ONLY write the three opening equations necessary to solve the problem: (then complete table)

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<td>K_{rot} (J)</td>
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<table>
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</table>
16.) In the figure below, the center of mass is clearly not over the pivot point. Therefore, the resulting torque should cause the top to fall over. Why doesn’t this happen? Label the following vectors to help you explain this phenomenon: \( mg, r, n, \tau, \) and \( L \).

17.) Complete the figure below showing the direction of \( L_i, L_f, dL, \tau, mg, \) and \( n \).

**40 pts from Chapter 11 homework problems:**

HW#1.) A ball of mass \( m \) is fastened at the end of a flagpole connected to the side of a tall building at point P, as shown in the figure below. The length of the flagpole is \( L \) and \( \theta \) is the angle the flagpole makes with the horizontal. Suppose that the ball becomes loose and starts to fall. Determine the angular momentum (as a function of \( t, \theta, L, g \) and \( m \)) of the ball about point P. Neglect air friction. (the Schluup short cut would come in handy here)
HW#2.) A spacecraft is in empty space. It carries on board a gyroscope with a moment of inertia of $I_g = 25 \text{ kg} \cdot \text{m}^2$ about the axis of the gyroscope. The moment of inertia of the spacecraft around the same axis is $I_s = 7.0 \times 10^5 \text{ kg} \cdot \text{m}^2$. Neither the spacecraft nor the gyroscope is originally rotating. The gyroscope can be powered up in a negligible period of time to an angular speed of $120 \text{ rad/sec}$. If the orientation of the spacecraft is to be changed by $34^\circ$, for how long should the gyroscope be operated?

HW#3.) A puck of mass 150 g is attached to a cord passing through a small hole in a frictionless, horizontal surface (see figure below). The puck is initially orbiting with speed of 15 m/s in a circle of radius 50.0 cm. The cord is then slowly pulled from below, decreasing the radius of the circle to 20.0 cm.

a.) Determine the tension (when $r = 20.0 \text{ cm}$) and
b.) Determine the work done in moving the mass from a radius of 50.0 cm to 20.0 cm.
HW#4.) A 150 kg uniform horizontal disk of radius 5.00 m turns without friction at 120 rpm on a vertical axis through its center. A feedback mechanism senses the angular speed of the disk and a drive motor at A maintains a constant angular speed while a 1.40 kg block starts at the center of the disk at t = 0 and moves outward with a constant speed of 11.9 mm/s relative to the disk until it reaches the edge in 7.0 minutes. The sliding block feels no friction. Its motion is constrained to have constant radial speed by a brake at B, producing a tension in a light string tied to the block. Find the work done by the drive motor during the block’s journey from starting point to the edge.

HW#5.) A solid cube of wood of side 4a and mass M is resting on a horizontal surface. The cube is constrained to rotate about an axis AB. A bullet of mass m and speed v is shot at the face opposite ABCD at a height of 8a/3. The bullet becomes embedded in the cube. Find the minimum value of v required to tip the cube so that it falls on face ABCD. Assume m << M