Chapter 10
Rotation of a Rigid Object About a Fixed Axis

Moment of inertia

\[ I = \lim_{\Delta m_i \to 0} \sum_{i} r_i^2 \Delta m_i = \int r^2 \, dm \]

Rectangular plate

\[ I_{CM} = \frac{1}{12} M(a^2 + b^2) \]

Long thin rod with rotation axis through end

\[ I = \frac{1}{3} ML^2 \]

Long thin rod with rotation axis through center

\[ I_{CM} = \frac{1}{12} ML^2 \]

Solid cylinder or disk

\[ I_{CM} = \frac{1}{2} MR^2 \]

Hoop or cylindrical shell

\[ I_{CM} = MR^2 \]

Solid sphere

\[ I_{CM} = \frac{2}{5} MR^2 \]

Thick spherical shell

\[ I_{CM} = \frac{2}{3} MR^2 \]

Hollow cylinder

\[ I_{CM} = \frac{1}{2} M(R_1^2 + R_2^2) \]

Rotational kinetic energy

\[ K_R = \frac{1}{2} I \omega^2 \]
10.1 During a certain period of time, the angular position of a swinging door is described by $\theta = 5.00 + 10.0t + 2.00t^2$, where $\theta$ is in radians and $t$ is in seconds. Determine the angular position, angular speed, and angular acceleration of the door (a) at $t = 0$ (b) at $t = 3.00$ s.

(b) $\alpha = 4 \text{ rad}^2/s^2$

(b) $\omega = 22 \text{ rad/s}$

4. An airliner arrives at the terminal, and the engines are shut off. The rotor of one of the engines has an initial clockwise angular speed of 2 000 rad/s. The engine's rotation slows with an angular acceleration of magnitude 80.0 rad/s$^2$. (a) Determine the angular speed after 10.0 s. (b) How long does it take the rotor to come to rest?

(a) $\omega_f = 1200 \text{ rad/s}$

(b) $t = 25.0 \text{ s}$

10.1.2 The tub of a washer goes into its spin cycle, starting from rest and gaining angular speed steadily for 8.00 s, at which time it is turning at 5.00 rev/s. At this point the person doing the laundry opens the lid, and a safety switch turns off the washer. The tub smoothly slows to rest in 12.0 s. Through how many revolutions does the tub turn while it is in motion?

50.0 rev

10.1.3 A rotating wheel requires 3.00 s to rotate through 37.0 revolutions. Its angular speed at the end of the 3.00-s interval is 98.0 rad/s. What is the constant angular acceleration of the wheel?

$\alpha = 13.7 \text{ rad/s}^2$

10.1.4 A wheel 2.00 m in diameter lies in a vertical plane and rotates with a constant angular acceleration of 4.00 rad/s$^2$. The wheel starts at rest at $t = 0$, and the radius vector of a certain point $P$ on the rim makes an angle of 57.3° with the horizontal at this time. At $t = 2.00$ s, find (a) the angular speed of the wheel, (b) the tangential speed and the total acceleration of the point $P$, and (c) the angular position of the point $P$.

(a) $\omega_f = 8.00 \text{ rad/s}$

(b) $a = 64.1 \text{ m/s}^2, \theta = 3.58 \text{ rad}$
16. A car accelerates uniformly from rest and reaches a speed of 22.0 m/s in 9.00 s. If the diameter of a tire is 58.0 cm, find (a) the number of revolutions the tire makes during this motion, assuming that no slipping occurs. (b) What is the final angular speed of a tire in revolutions per second?

18. A car traveling on a flat (unbanked) circular track accelerates uniformly from rest with a tangential acceleration of 1.70 m/s². The car makes it one quarter of the way around the circle before it skids off the track. Determine the coefficient of static friction between the car and track from these data.

21. The four particles in Figure P10.21 are connected by rigid rods of negligible mass. The origin is at the center of the rectangle. If the system rotates in the xy plane about the z axis with an angular speed of 6.00 rad/s, calculate (a) the moment of inertia of the system about the z axis and (b) the rotational kinetic energy of the system.

![Figure P10.21](image)

22. Two balls with masses $M$ and $m$ are connected by a rigid rod of length $L$ and negligible mass as in Figure P10.22. For an axis perpendicular to the rod, show that the system has the minimum moment of inertia when the axis passes through the center of mass. Show that this moment of inertia is $I = \mu L^2$, where $\mu = mM/(m + M)$.

![Figure P10.22](image)

24. Figure P10.24 shows a side view of a car tire and its radial dimensions. The rubber tire has two sidewalls of uniform thickness 0.635 cm and a tread wall of uniform thickness 2.50 cm and width 20.0 cm. Suppose its density is uniform, with the value $1.10 \times 10^3$ kg/m³. Find its moment of inertia about an axis through its center perpendicular to the plane of the sidewalls.

![Figure P10.24](image)
25. A uniform thin solid door has height 2.20 m, width 0.870 m, and mass 23.0 kg. Find its moment of inertia for rotation on its hinges. Is any piece of data unnecessary?

32. The tires of a 1 500-kg car are 0.600 m in diameter, and the coefficients of friction with the road surface are \( \mu_s = 0.800 \) and \( \mu_k = 0.600 \). Assuming that the weight is evenly distributed on the four wheels, calculate the maximum torque that can be exerted by the engine on a driving wheel without spinning the wheel. If you wish, you may assume the car is at rest.

33. Suppose the car in Problem 32 has a disk brake system. Each wheel is slowed by the friction force between a single brake pad and the disk-shaped rotor. On this particular car, the brake pad contacts the rotor at an average distance of 22.0 cm from the axis. The coefficients of friction between the brake pad and the disk are \( \mu_s = 0.600 \) and \( \mu_k = 0.500 \). Calculate the normal force that the pad must apply to the rotor in order to slow the car as quickly as possible.

Section 10.7 Relationship between Torque and Angular Acceleration

35. A model airplane with mass 0.750 kg is tethered by a wire so that it flies in a circle 30.0 m in radius. The airplane engine provides a net thrust of 0.800 N perpendicular to the tethering wire. (a) Find the torque the net thrust produces about the center of the circle. (b) Find the angular acceleration of the airplane when it is in level flight. (c) Find the linear acceleration of the airplane tangent to its flight path.

38. A potter's wheel—a thick stone disk of radius 0.500 m and mass 100 kg—is freely rotating at 50.0 rev/min. The potter can stop the wheel in 6.00 s by pressing a wet rag against the rim and exerting a radially inward force of 70.0 N. Find the effective coefficient of kinetic friction between wheel and rag.

39. An electric motor turns a flywheel through a drive belt that joins a pulley on the motor and a pulley that is rigidly attached to the flywheel, as shown in Figure P10.39. The flywheel is a solid disk with a mass of 80.0 kg and a diameter of 1.25 m. It turns on a frictionless axle. Its pulley has much smaller mass and a radius of 0.230 m. If the tension in the upper (taut) segment of the belt is 135 N and the flywheel has a clockwise angular acceleration of 1.67 rad/s², find the tension in the lower (slack) segment of the belt.
The hour hand and the minute hand of Big Ben, the famous Parliament tower clock in London, are 2.70 m long and 4.50 m long and have masses of 60.0 kg and 100 kg, respectively. Calculate the total rotational kinetic energy of the two hands about the axis of rotation. (You may model the hands as long thin rods.)

43. In Figure P10.43 the sliding block has a mass of 0.850 kg, the counterweight has a mass of 0.420 kg, and the pulley is a hollow cylinder with a mass of 0.350 kg, an inner radius of 0.020 m, and an outer radius of 0.030 m. The coefficient of kinetic friction between the block and the horizontal surface is 0.250. The pulley turns without friction on its axle. The light cord does not stretch and does not slip on the pulley. The block has a velocity of 0.820 m/s toward the pulley when it passes through a photogate. (a) Use energy methods to predict its speed after it has moved to a second photogate, 0.700 m away. (b) Find the angular speed of the pulley at the same moment.

45. An object with a weight of 50.0 N is attached to the free end of a light string wrapped around a reel of radius 0.250 m and mass 3.00 kg. The reel is a solid disk, free to rotate in a vertical plane about the horizontal axis passing through its center. The suspended object is released 6.00 m above the floor. (a) Determine the tension in the string, the acceleration of the object, and the speed with which the object hits the floor. (b) Verify your last answer by using the principle of conservation of energy to find the speed with which the object hits the floor.

47. This problem describes one experimental method of determining the moment of inertia of an irregularly shaped object such as the payload for a satellite. Figure P10.47 shows a mass $m$ suspended by a cord wound around a spool of radius $r$, forming part of a turntable supporting the object. When the mass is released from rest, it descends through a distance $h$, acquiring a speed $v$. Show that the moment of inertia $I$ of the equipment (including the turntable) is $mr^2(2gh/v^2 - 1)$.  

Figure P10.47
49. (a) A uniform, solid disk of radius \( R \) and mass \( M \) is free to rotate on a frictionless pivot through a point on its rim (Fig. P10.49). If the disk is released from rest in the position shown by the blue circle, what is the speed of its center of mass when the disk reaches the position indicated by the dashed circle? (b) What is the speed of the lowest point on the disk in the dashed position? (c) Repeat part (a), using a uniform hoop.

\[ 2 \sqrt{3} \]
\[ 4 \sqrt{3} \]
\[ \sqrt{8} \]

The head of a grass string trimmer has 100 g of wound in a light cylindrical spool with inside diameter 3.00 cm and outside diameter 18.0 cm, as in Fig. P10.50. The cord has a linear density of 10.0 g/m. A single strand of the cord extends 16.0 cm from the outer edge of the spool. (a) When switched on, the trimmer speeds from 0 to 2500 rev/min in 0.215 s. (a) What average power is delivered to the head by the trimmer motor when it is accelerating? (b) When the trimmer is cutting grass, it spins at 2000 rev/min and the grass exerts an average tangential force of 7.65 N on the outer end of the cord, which is still at a radial distance of 16.0 cm from the outer edge of the spool. What is the power delivered to the head under load?

\[ P = 74.3 \text{ W} \]
\[ 901 \text{ W} \]

63. A bicycle is turned upside down while its owner repairs a flat tire. A friend spins the other wheel, of radius 0.381 m, and observes that drops of water fly off tangentially. She measures the height reached by drops moving vertically (Fig. P10.63). A drop that breaks loose from the tire on one turn rises \( h = 54.0 \) cm above the tangent point. A drop that breaks loose on the next turn rises 51.0 cm above the tangent point. The height to which the drops rise decreases because the angular speed of the wheel decreases. From this information, determine the magnitude of the average angular acceleration of the wheel. Hint: This is Energy Conservation + Blackman.